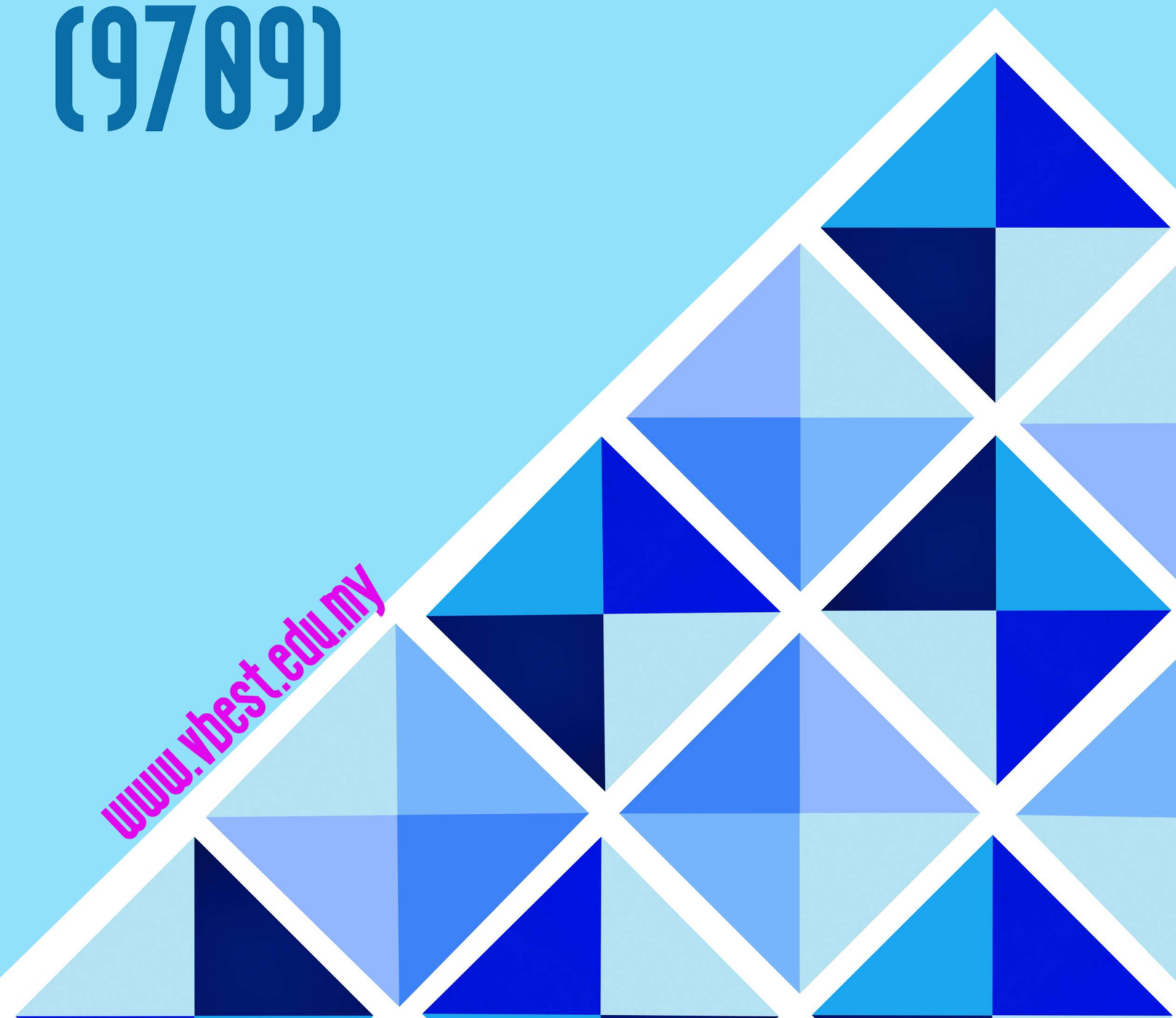
VBEST NOTES

A LEVEL CIE

AS PURE MATH 1

(9709)



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Math (Pure)

- ☐ Quadratics
- ☐ Functions
- ☐ Coordinate geometry
- ☐ Circular measure
- ☐ Trigonometry
- ☐ Vectors
- ☐ Series
- ☐ Differentiation
- ☐ Integration

Chapter 1: Quadratics

* Note: $(ax \pm b)^2 = (ax)^2 \pm 2abc \pm b^2$
NOT $(ax)^2 + bx^2$

a) Basic Algebra

- General form : $y = mx + c$
- Factorize a) $12x - 18y = 6(2x - 3y)$

  b) $6x + 5xy + 3x^2 = x(6 + 5y + 3x)$

b) Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Eg : $5x^2 - 4x - 1 = 0$

$$\frac{-(-4) \pm \sqrt{[(-4)^2 - 4(5)(-1)]}}{2(5)}$$

$= \frac{4 + \sqrt{36}}{10}$ $= \frac{4 - \sqrt{36}}{10}$

$= 1$ $= -0.2$

$\therefore x = 1$ or -0.2

c) Completing the square

Completing the square is where we take quadratic equations and turn them into

$(x + a)^2 + b$

$a(x + b)^2 + c$

—> $x^2 + b^2 + c = (x + b/2)^2 - (b/2)^2 + c$

—> $ax^2 + b^2 + c = a(x + b/2)^2 - a(b/2)^2 + c$

- Write in the form of $a(x + b)^2 + c$

a) $2x^2 - 5x$

$$2\left(x - \frac{5}{2}\right)^2 - \frac{25}{2}$$

b) $2x^2 - 3x - 4$

$$2\left(x - \frac{3}{2}\right)^2 - \frac{9}{2} - 4$$

- Write in the form of $(x + a)^2 + b$

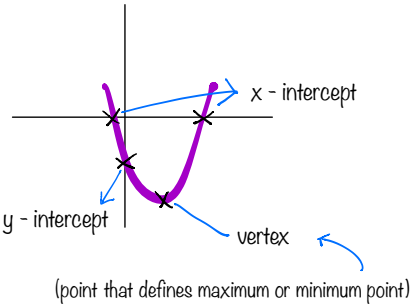
a) $x^2 + 4x$

$(x + 2)^2 + 4$

b) $6x^2 + 4x - 2$

$6(x + 2)^2 - 24 - 2$

d) Graph of quadratic functions



To identify x -intercept, y-intercept and vertex

Example : $-x^2-2x-13$

① Find y-intercept by equating x = 0

$-(0)^2- 2(0)+ 3 =0$

At y-axis, x = 0

$y = 3 \quad \therefore (0,3)$

② Find x- intercept by calculator or formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x^2 + 2x - 3 = 0$
$(x + 3)(x - 1) = 0$
$x = -3$ or 1

$\therefore (-3,0)$ & $(1,0)$

③ Use x = (-b)/(2a) to find x value of vertex

b and a are coefficients of x^2 and x

$x = \frac{-(-2)}{2(-1)} = -1$

④ Sub x value of vertex into f(x) to find y value of vertex

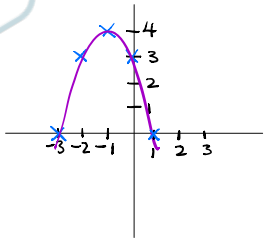
$f(-1) = -(-1)^2- 2(-1) + 3$
$= 4$

$\therefore (-1, 4)$

⑤ Choose another value to sub into f(x) to find 5th point

$f(-2) = -(-2)^2 -2(-2) + 3$
$= 3$

$\therefore (-2, 3)$

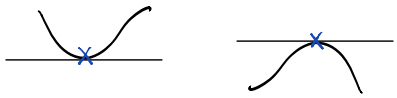


f) Discriminant

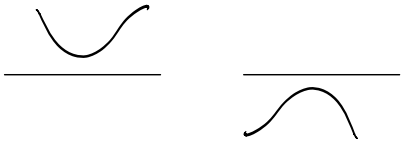
$b^2 - 4ac$ is called discriminant because it is the value that discriminates between the type of solution

$b^2 - 4ac < 0$ —> real & different roots

$b^2 - 4ac < 0$ —> equal roots



$b^2 - 4ac < 0$ —> no real roots



g) Solving simultaneous questions

* Always sub linear into quadratic!!

① Solve $x + 2y = 7$ and $x^2 + y^2 = 10$

$x = 7 - 2y$

$(7 - 2y)^2 + y^2 = 10$

$14 - 28y + 4y^2 + y^2 = 10$

$5y^2 - 28y + 4 = 0$

h) Solving inequalities

* Remember to flip sign when multiplying or dividing by a negative number

① Linear inequalities

Eg : $2x - 3 < 5$

$2x < 8$

$x < 4$

② Quadratic inequalities

→ solve quadratic eq.

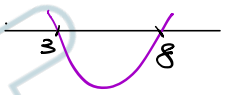
→ sketch graph

→ find required set

Eg : $x^2 - 11x + 24 < 0$

$(x - 3)(x - 8)$

$x = 3, x = 8$



$\therefore 3 < x < 8$

Chapter 2 : Functions

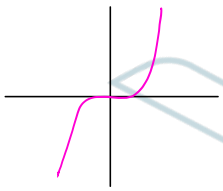
Domain: set of values of x

Range : set of values of y

a) Graph

(k is a positive constant)

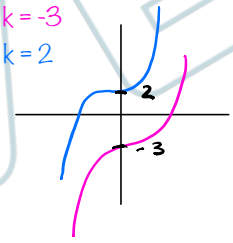
$y = bx^3$, $b > 0$



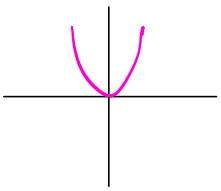
$y = x^3 + k$, $b < 0$

Eg : $k = -3$

Eg : $k = 2$

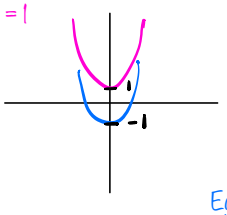


$y = bx^2$, $b > 0$



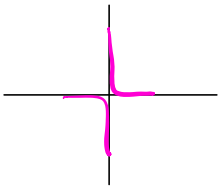
$y = bx^2 + c$, $b > 0$

Eg : $c = 1$



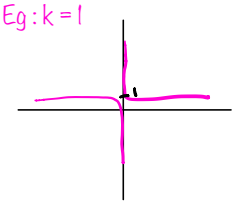
Eg : $c = -1$

$y = 1/x$, $b > 0$



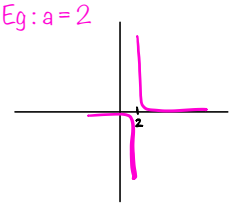
$y = 1/x + k$, $b > 0$

Eg : $k = 1$



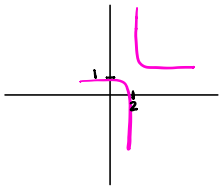
$y = b/(x-a)$, $a > 0$, $b > 0$

Eg : $a = 2$



$y = b/(x-a) + k$, $a > 0$, $b > 0$

Eg : $b = 1$, $a = 2$, $k = 1$



c) Inverse function

* For a function f(x) to have inverse function (f^{-1}), it must be one to one.
* Range of f(x) is domain of $f^{-1}(x)$ and vice versa
* Domain of f(x) is range of $f^{-1}(x)$ and vice versa
* Graph of $f^{-1}(x)$ is the reflection of f(x) along y=x
* To find $f^{-1}(x)$ of quadratic function, use completing the square

Eg : $g(x) = 2x - 3$, for $-2 \le x \le k$

i) State the largest value of k for which g is one to one (has an inverse)

$2x - 3 = 0$

$2x = 3$ $\quad k = 1.5$

$x = 1.5$

ii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}

$y = 2x - 3$

$\frac{y + 3}{2} = x$ $\quad f^{-1} = \frac{x + 3}{2}$

d) Composite function

- $g(x) = 5x$
- $f(x) = 2x - 13$

$\rightarrow ff(x) = 2(2x + 3) + 3$

$\rightarrow fg(x) = 2(5x) + 3$

$\rightarrow gf(x) = 5(2x + 3)$

Chapter 3: Coordinate geometry

Distance between 2 points $= \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Midpoint $= \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$

Gradient : $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation of a line $= y - y_1 = m(x - x_1)$

General equation : $y = mx + c$ (m: gradient, c: y - intercept)

If 2 lines are parallel: $m_1 = m_2$

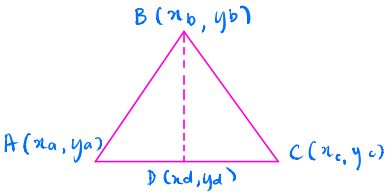
If 2 lines are perpendicular: $m_1 \times m_2 = -1$

Perpendicular bisector : Both lines are perpendicular and intersect at midpoint

a) Polygon

Area of a $\Delta ABC = (1/2)(AC)(BD)$

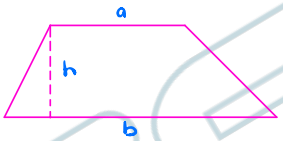
BD = distance of BD
AC = distance of AC



Area of polygons $= (1/2)\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$

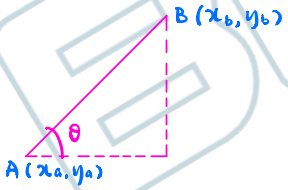
$= (1/2)\,|(x_1y_2 + x_2y_3 + x_3y_1) - (x_1y_3 + x_3y_2 + x_2y_1)|$

Area of trapezium $= \frac{(a+b)}{2} = h$



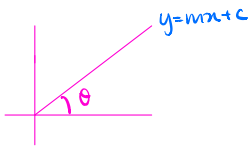
b) Angle between lines

$\tan\theta = \frac{y_1 - y_2}{x_1 - x_2} = m$

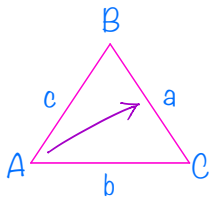


Since $\tan\theta = m$

$m = \tan^{-1}\theta$



Chapter 4 : Circular measure



a) Sine rule

Use if you know :

—> two angles and the length of one of their opposite side

—> the length of two sides and one of the opposite angle

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

b) Cosine

Use if you know :

—> lengths of two sides and the angle between them

—> the length of all 3 sides

$$a^2 = b^2 + c^2 - 2bc\cos A$$

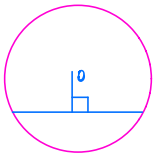
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

c) Area of triangle —> Area Δ = 1/2 ab sin C

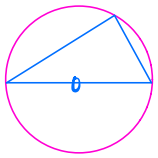
g) Area of a sector —> A = 1/2 r^2 θ

d) The length of an arc of a circle —> S = rθ

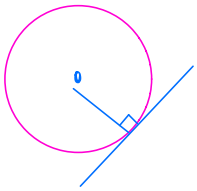
f) Properties of a circle



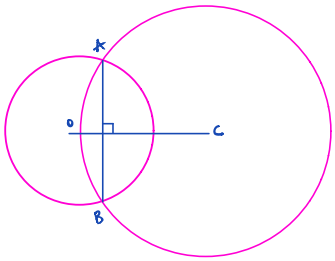
The perpendicular from the centre of a circle to chord bisects the chord.



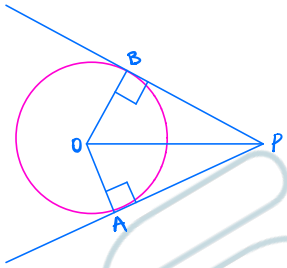
The angle in a semicircle is a right angle



A tangent is a line that meets the circle at one point only



The line joining the intersection point of two circles is perpendicular to the line joining the two centres

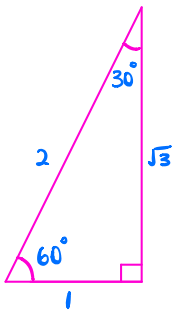


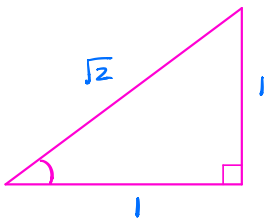
To tangent lines will form 2 congruent triangles

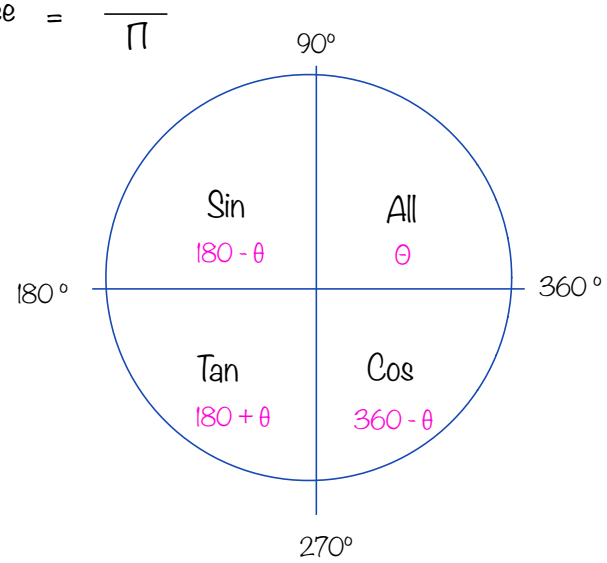
Chapter 5 : Trigonometry

a) Radian

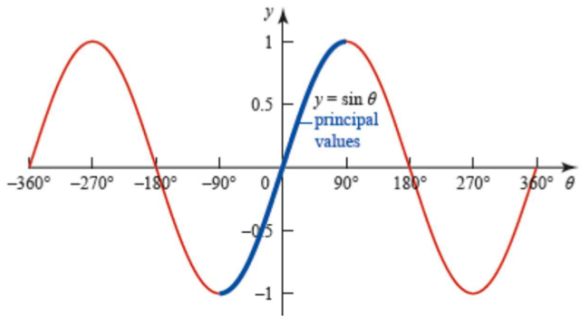
- 1 rad = $\frac{180^\circ}{\pi}$
- Π rad = 180°
- Degree to rad = $\frac{x^\circ}{180}$ x π radian
- Rad to degree = $\frac{x}{\Pi}$ x 180°



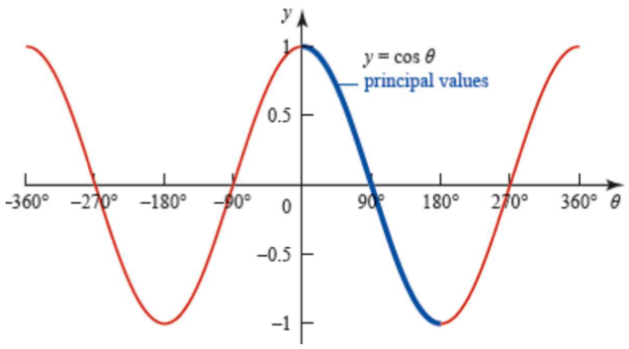




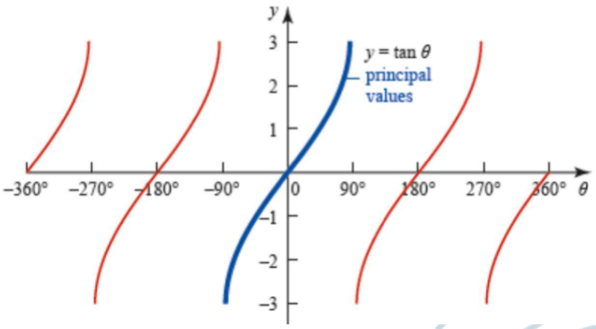
b) Graphs of sin θ, cos θ and tan θ



Sine graph



Cos graph



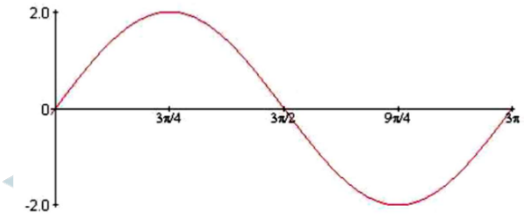
Tan graph

c) Identifying transformations of graph

$$y = a \sin(bx) + c$$

- a: amplitude
- b: alters number of cycle
- c: alters y-axis

Eg :



Find the values of a, and b.

$\therefore a = 2$

$\therefore b = 1/4$

d) Trigonometry identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Chapter 6 : Vector

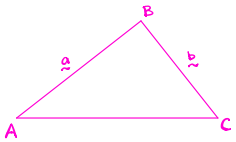
a) Notation of vector :  or 

b) Equal vector Two vectors AB and CD are equal if and only if :

- $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$
- $|\overrightarrow{AB}| = |\overrightarrow{CD}|$
- The direction from A to B is same as from C to D

c) Addition of vectors

To find AC you must add $\overrightarrow{AB}$ and $\overrightarrow{BC}$



$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

d) Unit vector

$$\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$$

Vector divided by magnitude of vector

e) Position vector = Position of vector relative to origin = $\overrightarrow{OA}$

Given $\overrightarrow{OA}$ and $\overrightarrow{OB}$, find $\overrightarrow{AB}$ Eg : $\overrightarrow{OA} : 2i + 5j$ $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{OB} : 3i + j$ $= \begin{pmatrix} 2i & - & 3i \\ 5j & - & j \end{pmatrix} = -i + 4j$

f) Distance

- Magnitude of vector $xi + yj + zk = \sqrt{(x^2 + y^2 + z^2)}$
- Distance between two points is $\sqrt{[(x_1 - x_2) + (y_1 - y_2) + (z_1 - z_2)]}$

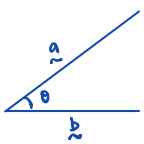
g) Midpoint Mid point of AB has position vector $1/2(\overrightarrow{OA} + \overrightarrow{OB})$

If M is the midpoint of AB then $\overrightarrow{OM} = 1/2(\overrightarrow{OA} + \overrightarrow{OB})$ and not $1/2\ \overrightarrow{AB}$

h) Scalar product (Dot product)

$a \cdot b = |a|\,|b| \cos\theta$

$\therefore \cos\theta = \dfrac{a \cdot b}{|a|\,|b|}$



*Note : If two vectors are perpendicular, a.b = 0

Use scalar product to find angle between two vectors

Chapter 7 : Series

Arithmetic : General term : $U_n = a + (n-1)d$ (a: First term, d: Common difference)

Sum of arithmetic series

$S_n = (n/2)[2a + (n-1)d]$

or

$S_n = (n/2)(a + L)$

Geometric : General term : $a_n = ar^{n-1}$

Sum off geometric series

$S_n = \frac{a(r^n - 1)}{r - 1}$, $r > 1$ $\qquad$ $S_n = \frac{a(1 - r^n)}{1 - r}$, $r < 1$

(r: Common ratio)

Sum to infinity : $S_\infty = \frac{a}{1 - r}$

a) Binomial expansion

$(a + b)^n = ({}^nC_0)(a^n) + ({}^nC_1)(a^{n-1})(b) + ({}^nC_2)(a^{n-2})(b^2)\ldots\ldots$

$(1 + x)^n = 1 + nx + \frac{n(n-1)(x)^2}{2!} + \frac{n(n-1)(n-2)(x^3)}{3!} \ldots\ldots\ldots\ldots$

Must always be 1

Chapter 8 : Differentiations

$\frac{dy}{dx}$ or $f'(x)$

a) Differentiations

Eg : $y = 2x^3$

$\frac{dy}{dx} = 6x^2$

*Change $\sqrt{x}$ and $(1/x)$ to $x^{\frac{1}{2}}$ and x^{-1} respectively

b) Chain rule

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

*Note : Differentiate outside then differentiate inside

Eg : $f(x) = (1 + 2x)^3$ $\quad f'(x) = 3(1 + 2x)^2 \times 2$

$= 6(1 + 2x)^2$

c) Second derivatives (Differentiate twice) $\quad \frac{d^2y}{dx^2}$ or $f''(x)$

d) Gradient of the curve

$\frac{dy}{dx}$ = gradient of the curve

Eg : Find the gradient of the curve $f(x) = 3x^2$ at point (2,12)

$f'(x) = 6x$, therefore gradient is $6(2) = 12$

e) Increasing, decreasing and stationary function

Increasing = $f'(x) > 0$

Decreasing = $f'(x) < 0$

Stationary/Turning point = $f'(x) = 0$

f) Nature of the graph

Use second derivative to find nature of graph

Maximum : $f''(x) < 0$

Minimum : $f''(x) > 0$

g) Rate of change of a function

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

Eg :

The equation of the curve is $y=x^2 - 5x$

A point P is moving along the curve so that the x coordinate is increasing at a constant rate of 0.2 units per second. Find the rate at which the y coordinate is increasing when $x = 4$

$2x - 5 = (dy/dt) \times (1/0.2)$

$2(4) - 5 = (dy/dt) \times 5$

$0.6 = dy/dt$

Formulas for volume of :

Sphere = $(4/3)\pi r^3$

Cone = $\pi r^2 (h/3)$

Pyramid = $LWH/3$

Cylinder = $\pi r^2 h$

Chapter 9 : Integration

$c + x^2 \xrightarrow{\text{differentiation}} 2x$

integration

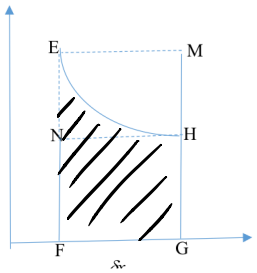
$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

a) Integration

Eg : $f'(x) = 2x^2 + 3$

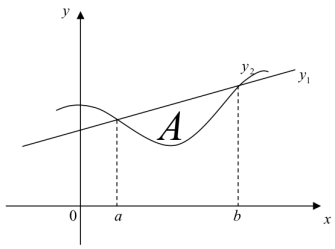
$f(x) = \frac{2x^3}{3(2)} + 3x$

b) Area under the curve



Integrate from upper limit (G) to lower limit (F)

c) Area between a line and a curve



$$\text{Area} = \int_a^b y_1\,dx - \int_a^b y^2\,dx \quad \text{(upper bound – lower bound)}$$

d) Volume of revolution

About the x-axis = $\pi \int_a^b y^2\,dx$

About the y-axis = $\pi \int_a^b x^2\,dy$

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